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BOOK OF ABSTRACTS

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Preface

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Contents

Banach limits, Banach measure, and properties of spaces of integrable functions

E. A. Dzhenzher [1],

Keywords: Banach limit, Banach measure, integral over a Banach measure, Fourier transform

MSC2020 codes: 28A12

Introduction. As is well known, there is no translation-invariant, finite, sigma-additive measure on a real axis. We will abandon sigma-additivity and discuss the construction of a family of Banach measures obtained using the ultrafilter limit. We will construct an integral over a Banach measure and discuss the Fourier transform over it. In addition, we will discuss the properties of L_p spaces of functions integrable over the Banach measure.

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On approximations of semigroup C^*-algebras

R. N. Gumerov [2], E. V. Lipacheva [3].

Keywords: inductive limit; semigroup C^*-algebra; universal C^*-algebra.

MSC2020 codes: 46L05, 46M40, 47L40

Introduction. In the talk, we consider the left reduced semigroup C^*-algebras $C^*_\lambda(\mathcal{Q}_{\overline{N}})$ for semigroups $\mathcal{Q}_{\overline{N}}$ which are the free products for infinite families of semigroups of rational numbers. Such free products are constructed using countable collections $\overline{N} := (N_1, N_2, \ldots)$, where N_i are arbitrary sequences of natural numbers for all $i \in \mathbb{N}$.

The main result is concerned with an approximation of the C^*-algebras $C^*_\lambda(\mathcal{Q}_{\overline{N}})$ by inductive sequences consisting of copies for the universal C^*-algebra $\mathcal{O}_\infty$ generated by a countable family of isometries with pairwise orthogonal ranges. As corollaries of this result, we get that the C^*-algebras $C^*_\lambda(\mathcal{Q}_{\overline{N}})$ are simple and prove that the C^*-algebra $\mathcal{O}_\infty$ coincides up to an isomorphism with the C^*-algebra generated by the representation of the free product for the countable family of copies for the additive semigroup of all natural numbers.

A motivation for our work came from the results on properties of C^*-algebras and their morphisms in [1-3] which were obtained using approximations by inductive sequences consisting of universal C^*-algebras generated by isometries.

Main result.

Theorem. Let $\overline{N} = (N_1, N_2, \ldots)$ be a countable collection of sequences of natural numbers. Then the C^*-algebra $C^*_\lambda(\mathcal{Q}_{\overline{N}})$ is the limit of the inductive sequence consisting of the copies of the Cuntz algebra $\mathcal{O}_\infty$ and its $*$-endomorphisms.

Corollary. The C^*-algebra $C^*_\lambda(\mathcal{Q}_{\overline{N}})$ is simple.

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Markov operators and quantum channels

S. G. Haliulin [4]

Keywords: Markov operators; ergodicity; quantum channels.

MSC2020 codes: 81Q10

Introduction. The report will consider Markov operators operating on a probability gauge space with an exact trace state specified on it. The Markov quantum channels will be defined and its ergodic properties will be considered.

Let $(H, \mathcal{M}, m)$ be a probability gauge space (see, for example, [1]), that is, $\mathcal{M}$ is a von Neumann algebra on a Hilbert space H with a faithful normal tracial state m. Naturally, the scalar product is introduced on the algebra $\mathcal{M}$: $(x, y) = \tau(y^*x)$. For $x \in \mathcal{M}$, we put $\|x\|^2 = \tau(x^*x)$. The completion of the algebra $\mathcal{M}$ with respect to this norm forms a Hilbert space denoted by $L^2(\mathcal{M}, \tau)$.

Definition 1. (compare, [2]) Let's define a probability gauge space $(H, \mathcal{M}, m)$. The Markov operator defined in the Hilbert space $L^2(\mathcal{M}, \tau)$ is the linear bounded operator $T : L^2(\mathcal{M}, \tau) \to L^2(\mathcal{M}, \tau)$, satisfying the following conditions: 1) $\|T\| \le 1$; 2) $T(\mathbf{1}) = T^*(\mathbf{1}) = \mathbf{1}$; 3) the operator T preserves positivity, that is, Tx will be a positive element of $L^2(\mathcal{M}, \tau)$ if x is a positive element of $L^2(\mathcal{M}, \tau)$.

Definition 2. A linear operator T operator in the space $L^2(\mathcal{M}, \tau)$ is said to be ergodic if for any $x, y \in L^2(\mathcal{M}, \tau)$, $\mathbf{0} < x < \mathbf{1}$, $\mathbf{0} < y < \mathbf{1}$ there exists a $n \in \mathbb{Z}^+$ such that $(T^n x, y) > 0$.

Definition 3. Let T_1 and T_2 be two different self-adjoint Markov operators acting on $L^2(\mathcal{M}, \tau)$. Let's say that the Markov operators T_1 and T_2 are equivalent if $(T_1x, y) = 0$ implies $(T_2x, y) = 0$, $x, y \in L^2(\mathcal{M}, \tau)$, and vice versa; Markov operators T_1 and T_2 are mutually singular if there exist such $x, y \in L^2(\mathcal{M}, \tau)$ such that $(T_1x, y) = 0$, $(T_2x, y) = 1$.

Theorem 1. Let T_1 and T_2 be two different self-adjoint Markov ergodic operators operating in the space $L^2(\mathcal{M}, \tau)$. Then they are either equivalent or singular.

Theorem 2. Let T be a self-adjoint Markov ergodic operator in the space $L^2(\mathcal{M}, \tau)$, where $(H, \mathcal{M}, \tau)$ is a probability gauge space. Then there exists a unique positive operator P such that

$$\frac{1}{N} \sum_{n=0}^{N-1} T^n \xrightarrow{L^2} P,\ N \to \infty,$$

and P is a projector onto the eigenspace of the operator T, corresponding to the eigenvalue $\lambda = 1$, λ has multiplicity one.

These and other results are easily transferred to Markov quantum channels.

Let's denote the convex set of density operators (states) by $\mathcal{S}(L^2)$.

Definition 4. Let's call the Markov self-adjoint completely positive operator $T : \mathcal{S}(L^2) \to \mathcal{S}(L^2)$ a Markov quantum channel.

The properties of Markov quantum channels will be discussed in the report.

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Concrete examples of the rate of convergence of Chernoff approximations: numerical results for the heat semigroup and open questions on them

K. A. Katalova(Dragunova),[5] I. D. Remizov,[6] N. Nikbakht[7]

Keywords: operator semigroups, Chernoff approximations, convergence rate, numerical study, heat equation, initial value problem.

MSC2020 codes: 65M12, 47D06, 35K05, 35E15, 35C99

Abstract. The talk is devoted to the article that discusses the construction of examples that illustrate (using computer calculations) the rate of convergence of Chernoff approximations to the solution of the Cauchy problem for the heat equation. We are interested in the Chernoff theorem [1] in general and select the heat semigroup as a model case because this semigroup (and solutions of the heat equations) are known, so it is easy to measure the distance between the exact solution and its Chernoff approximations. Two Chernoff functions [2,3] (of the first and second order of Chernoff tangency to the generator of the heat semigroup, i.e. to the operator of taking the second derivative) and several initial conditions of different smoothness are considered. From the numerically plotted graphs, visually, it is determined that the approximations are close to the solution. For each of the two Chernoff functions, for several initial conditions of different smoothness and for approximation numbers up to 11 inclusive, the error (i.e. the supremum of the absolute value of the difference between the exact solution and the approximating function) corresponding to each approximation was numerically found. As it turned out, in all the cases studied, the dependence of the error on the number of the approximation has an approximately power-law form (we call this power the order of convergence). This follows from the fact that, as we discovered, the dependence of the logarithm of the error on the logarithm of the approximation number is approximately linear. Using the considered family of initial conditions, an empirical dependence of the order of convergence on the smoothness class of the initial condition is found. The orders of convergence for all the initial conditions studied are collected in a table. We also found some behavior of the error that we summarize in the conclusion and that we do not know how to explain with the existing theory, which opens a field of work for further theoretical research.

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Quantization of field and evolution in the physically general case

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Keywords: field quantization; quantum evolution; Hamiltonian; coherent states.

MSC2020 codes: 81T05, 81S05, 81T70, 81V10

This report outlines the Bose quantization of a classical Hamiltonian system by imposing only **physically general** constraints on the Hamiltonian:

(i) Positive definiteness (the energy of a system can be measured relative to vacuum energy).

(ii) Finite values of energy and momentum (as conserved quantities).

(iii) Continuity of momentum (if the system is perturbed slightly so that its energy changes only slightly, then the system's momentum must also change only slightly).

(iv) Thermodynamic properties in a bounded domain: a finite value of the Gibbs trace and completeness of the eigenbasis.

(v) Galilean (Lorentz) invariance: in the limit of small linear scales, a particle behaves as free (other particles are far away, and external fields are approximately constant at such scales).

(vi) Finally, some continuity properties (intrinsically related to the impossibility of exact measurement in an experiment), as well as homogeneous boundary conditions of the first kind, the identity principle, probability normalization, and the superposition principle.

The quantization of the field and its evolution is based on the exact equality of energy (and similarly for momentum) in the classical and quantum cases on **coherent** states:

$$H_{\text{cl}}[\alpha(\mathbf{r})] = H[|\alpha\rangle] \text{ and } \mathbf{P}_{\text{cl}}[\alpha(\mathbf{r})] = \mathbf{P}[|\alpha\rangle] \qquad (\forall \alpha(\mathbf{r}) \in D[H_{\text{cl}}]). \tag{1}$$

Here, $|\alpha\rangle$ is a coherent state vector in Fock space ($\hat{\psi}(\mathbf{r})|\alpha\rangle = \alpha(\mathbf{r})|\alpha\rangle$; $\hat{\psi}(\mathbf{r})$ is the field operator), $D[A]$ is the domain of the quantity A, $H[|\psi\rangle]$ is the quantum Hamiltonian functional in Fock space ($H[|\psi\rangle] = \langle\psi|\hat{H}|\psi\rangle$, $\hat{H}$ is the quantum Hamiltonian operaror), $\mathbf{P}[|\psi\rangle]$ is the momentum functional ($\mathbf{P}[|\psi\rangle] = \langle\psi|\hat{\mathbf{P}}|\psi\rangle$, with $\hat{\mathbf{P}} = \int d\mathbf{r}\hat{\psi}^\dagger(\mathbf{r})(-i\hbar\nabla)\hat{\psi}(\mathbf{r})$), $\mathbf{r} \in \mathbb{R}^{\text{dim}}$ is the spatial coordinate (typically, $\text{dim} = 3$), and $H_{\text{cl}}[\psi(\mathbf{r})]$ and $\mathbf{P}_{\text{cl}}[\psi(\mathbf{r})]$ are the corresponding classical energy and momentum functionals. These can always be expressed through the (classical) field $\psi(\mathbf{r})$, provided we express (a finite number of) classical fields $\mathbf{A}(\mathbf{r})$, on which energy and momentum depend in the classical theory, in terms of $\psi(\mathbf{r})$:

$$H_{\text{cl}}[\psi(\mathbf{r})] = H_{\text{CL}}\left[\breve{\mathbf{A}}[\psi(\mathbf{r})]\right], \quad \mathbf{P}_{\text{cl}}[\psi(\mathbf{r})] = \mathbf{P}_{\text{CL}}\left[\breve{\mathbf{A}}[\psi(\mathbf{r})]\right], \quad \mathbf{A}(\mathbf{r}) = \breve{\mathbf{A}}[\psi(\mathbf{r})], \tag{2}$$

where $\breve{\mathbf{A}}$ is an integral convolution (its linearity is not required), and $H_{\text{CL}}[\mathbf{A}(\mathbf{r})]$ and $\mathbf{P}_{\text{CL}}[\mathbf{A}(\mathbf{r})]$ represent the dependence of energy and momentum in the classical theory on their original fields $\mathbf{A}(\mathbf{r})$. For simplicity, the spinless case is described.

With this quantization method, the second condition in (1) imposes a constraint on the form of the integral convolution $\breve{\mathbf{A}}$, i.e., on the expressions for the fields $\hat{\mathbf{A}}$ in terms of $\hat{\psi}(\mathbf{r})$. The first condition in (1) determines the specific form of the quantum Hamiltonian functional $H[|\psi\rangle]$, or, in other words, the quantum evolution. The remaining arbitrariness in the form of the integral convolution $\breve{\mathbf{A}}$ (partially or totally) is eliminated by the necessity to satisfy the axiomatic requirements for the Hamiltonian functional $H[|\psi\rangle]$, which follow from postulates (i)-(vi). If the integral convolution cannot be found, then the classical system cannot be quantized.

The quantization described by (1) always yields a **normal** ordering of the fields $\hat{\psi}(\mathbf{r})$ and $\hat{\psi}^\dagger(\mathbf{r})$. As a result, the first divergence in quantum electrodynamics vanishes automatically. Moreover, the quantization of nonlinear terms such as higher powers of the electric (and magnetic) fields also proves to be divergence-free, which is a notable outcome. Finally, the Casimir effect is preserved if it is interpreted as an interaction of charges in the photon vacuum mediated by virtual photons.

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Weak coupling limit for quantum systems with unbounded weakly commuting system operators

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Keywords: Quantum Theory; Reduced Dynamic.

MSC2020 codes: 81S22, 81Q15

The theory of open quantum systems studies the dynamics of quantum systems interacting with external environments, or reservoirs. Since most realistic physical systems are not completely isolated, a central problem in this field is the derivation of equations governing the reduced dynamics of the system alone, after eliminating the reservoir degrees of freedom under physically motivated approximations. One of the widely used frameworks is the weak coupling limit (WCL), which captures the long-time behavior of a quantum system weakly interacting with a environment.

The weak coupling limit is well studied for finite-dimensional systems. In contrast, much less is known for infinite-level systems, especially in the presence of unbounded system Hamiltonians and continuous spectra. In this work, we address this problem for an infinite-level quantum system interacting with an ideal Bose or Fermi gas. We assume that the free system Hamiltonian H_S commutes, in a weak sense, with the system components Q_j of the interaction operator $V = \sum_j Q_j \otimes a^+(f)a(g)$, without imposing restrictions on the spectral properties of H_S. The full system-reservoir dynamics is generated by the Heisenberg equation with coupling constant λ. The reduced dynamics is obtained by partial trace $\mathring{\omega}$ over the reservoir with respect to a fixed stationary state ω, i.e. $\mathring{\omega}(A \otimes B) := A \cdot \omega(B)$. The weak coupling limit is defined by the scaling $\lambda \to 0$ and $t \to \infty$, such that the rescaled time $\lambda^2 t$ remains finite.

A key step of the analysis is the study of multi-point correlation functions of the reservoir in the weak coupling limit

$$\Omega_\pi(\lambda) := \lambda^k \int\limits_{\{\lambda^{-2}t>t_1>t_2>\cdots>t_k>0\}} \omega\Big(a^\#(e^{it_{\pi(1)}h} f_1) a^\#(e^{it_{\pi(2)}h} f_2) \dots a^\#(e^{it_{\pi(k)}h} f_k)\Big) dt_1 \dots dt_k,$$

where π is a permutation of k elements and $a^\#(e^{ith} f)$ denotes the free evolution of the creation or annihilation operator in the interaction picture. We derive a recursive formula for the limiting behavior of these correlation functions as $\lambda \to 0$ and provide explicit convergence estimates. These results allow us to analyze its asymptotic behavior of Dyson series terms of the reduced system dynamics.

Using this approach, we prove that the reduced dynamics converges, in the weak coupling limit, to a unitary evolution generated by an effective Hamiltonian. This Hamiltonian consists of the original system Hamiltonian H_S and an additional correction term interpreted as a Lamb shift induced by the interaction with the reservoir. We obtain an explicit expression for this Lamb shift and show that the convergence to the limiting dynamics occurs with rate $O(\lambda)$.

Acknowledgments. The work is supported by Ministry of Science and Higher Education of the RF, grant No 075-15-2024-529

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On minimal sets of continuous maps on finite trees and dendrites

E. N. Makhrova [11].

Keywords: finite tree, dendrite, minimal set, topological entropy.

MSC2020 codes: 37B40; 37B45; 37E25; 54F50.

One of the main questions in the theory of dynamical systems is the question of the existence of minimal sets and their relationship with the properties of a dynamical system. Results of this type of research on continuous maps of an interval or finite trees are mainly related to finite minimal sets, that is, periodic orbits. For example, a connection has been established between the periods of periodic points, topological entropy, and the properties of a continuous map of an interval (see, for example, [1]). In [2], conditions are obtained for the periodic orbits of a continuous map of a finite tree under which the topological entropy of the given map is positive. It should be noted that the properties of both finite and infinite minimal sets for a continuous map of an interval and 3-od were studied in [3], [4], using the concept of a D-function of a minimal set. In [5], the properties of infinite minimal sets for a continuous map of an interval with zero topological entropy are described. As for continuous maps of dendrites, finite minimal sets do not affect topological entropy [6], but there is a connection between the properties of infinite minimal sets and topological entropy [7].

In the report we study the properties of infinite minimal sets for continuous maps with various entropy characteristics defined on one-dimensional branched continua such as finite trees and dendrites (see [8], [9]).

The work was supported by the Russian Science Foundation (grant no. 24-21-00242).

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Analysis of spectral properties of a general fourth–order operator with a parameter in the boundary condition

D. M. Polyakov [12]

Keywords: eigenvalue asymptotics; spectral parameter in the boundary conditions; fourth–order differential operator.

MSC2020 codes: 34B08, 34L20,

We consider the following spectral problem

$$y^{(4)}(x) - (q(x)y'(x))' = \lambda y(x), \tag{1}$$

$$y''(0) - a\lambda y'(0) = 0, \quad \mathcal{T}y(0) - b\lambda y(0) = 0, \quad y''(1) - c\lambda y'(1) = 0, \quad \mathcal{T}y(1) - d\lambda y(1) = 0, \tag{2}$$

where $\lambda \in \mathbb{C}$ is a spectral parameter, $q \in W_1^3(0,1)$ is real function, $\mathcal{T}y = y''' - qy'$, and $a < 0$, $b > 0$, $c > 0$, $d < 0$ are real constants. We can reduce (1) and (2) to a spectral problem for some linear operator $\mathcal{L}$. We denote the eigenvalues of this operator by λ_n.

The main purpose of this report is to determine a sharp formula for the asymptotics of λ_n. Our work is closely related to the article [1]. In this paper Aliyev and Fleydanli studied a localization of the spectrum and basis property for the system of eigenfunctions and associated functions in $L_p(0,1)$, $1 < p < \infty$. Moreover, they obtained the following rough asymptotics

$$\lambda_n = \pi^4\big(n - 7/2\big)^4 + \pi^2\big(n - 7/2\big)^2\big(q_0 + 4\varkappa\big) + \mathcal{O}(n^{-1}),$$

as $n \to +\infty$, where q is absolutely continuous function and

$$q_0 = \int_0^1 q(x)\,dx, \qquad \varkappa = 1/b - 1/d. \tag{3}$$

In this report we improve this asymptotics (with additional smoothness of q).

We formulate our main result.

Theorem 1. Suppose that $q \in W_1^3(0,1)$ and $q(0) = q(1)$. Then the eigenvalues λ_n of the problem (1) and (2) satisfy the following asymptotics

$$\lambda_n = \pi^4\Big(n - \frac{7}{2}\Big)^4 + \pi^2\Big(n - \frac{7}{2}\Big)^2\big(q_0 + 4\varkappa\big) - 2\pi\Big(n - \frac{7}{2}\Big)\big(q(0) + 2\mathcal{E}\big) + \varkappa q(0) + \frac{q_0^2 - \|q\|^2}{8}$$
$$+ 2\varkappa^2 + \varkappa q_0 + \frac{4}{c} - \frac{4}{a} + \frac{8}{3}\Big(\frac{1}{b^3} - \frac{1}{d^3}\Big) + \frac{1}{\pi(n - 7/2)}\Big(4\gamma_n - \frac{\widehat{q}'''_{cn}}{8}\Big) + \mathcal{O}(n^{-2}),$$

as $n \to +\infty$, where $\varkappa$ has the form (3) and

$$\mathcal{E} = \frac{1}{b^2} + \frac{1}{d^2}, \qquad \widehat{q}'''_{cn} = \int_0^1 q'''(x)\cos\pi(2n-7)x\,dx, \qquad n \in \mathbb{Z},$$

and

$$\gamma_n = \frac{1}{32}\int_0^1 e^{-\pi(2n-7)s}\big(p'''(s) - p'''(1-s)\big)\,ds$$
$$+ \frac{1}{4}\int_0^1 e^{-\pi(n-7/2)s}\big(p'''(s) - p'''(1-s)\big)\sin\pi(n - 7/2)s\,ds.$$

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Convolution operators in the Roumieu spaces of ultradifferentiable functions and ultradistributions

D. A. Polyakova [13]

Keywords: ultradifferentiable functions; ultradistributions; convolution operator

MSC2020 codes: 44A35, 46E10

We consider the convolution operators T_μ and S_μ acting in the Roumieu space $\mathcal{E}^p_{\{\omega\}}(\mathbb{R})$ of ultradifferentiable functions of mean type and in the corresponding space $\left(D^p_{\{\omega\}}(\mathbb{R})\right)'$ of ultradistributions. Here ω is a weight function determining the spaces; $p \in (0, \infty)$ is a type of the spaces; μ is a symbol of convolution operators, i. e. an entire function with certain growth conditions. As particular cases, the operators T_μ include the differential operators of infinite order with constant coefficients, differential-difference and integro-differential operators.

We firstly obtain a criterion of surjectivity of operators T_μ and S_μ. Then we get a criterion when surjective operators T_μ and S_μ admit continuous linear right inverse operators. Necessary and sufficient conditions are formulated in terms of the null set $N(\mu) = (\lambda_j)_{j=1}^\infty$ of symbol μ. The main results are two following theorems.

Theorem 1. 1) The convolution operator T_μ in $\mathcal{E}^p_{\{\omega\}}(\mathbb{R})$ is surjective if and only if two following assumptions are satisfied:

$$(i) \quad \forall \varepsilon > 0 \;\; \forall \delta > 0 \;\; \exists r_0 > 0 \mid \forall x \in \mathbb{R} \text{ with } |x| \geq r_0 \;\; \exists t \in \mathbb{R} \text{ with } |t| > |x| : \\ |t - x| \leq \delta\omega(x) \;\text{ and }\; |\mu(t)| \geq \exp\{-\varepsilon\omega(t)\}\,;$$

$$(ii) \quad N(\mu) = N_1 \cup N_2 : \quad \lim_{j\to\infty, \lambda_j \in N_1} \frac{|\operatorname{Im}\lambda_j|}{\omega(\operatorname{Re}\lambda_j)} = 0\,; \quad \liminf_{j\to\infty, \lambda_j\in N_2} \frac{|\operatorname{Im}\lambda_j|}{\omega(\operatorname{Re}\lambda_j)} > 0\,.$$

2) S_μ is surjective in $\left(D^p_{\{\omega\}}(\mathbb{R})\right)'$ if and only if condition (i) holds.

Theorem 2. Let T_μ and S_μ be surjective convolution operators in $\mathcal{E}^p_{\{\omega\}}(\mathbb{R})$ and $\left(D^p_{\{\omega\}}(\mathbb{R})\right)'$. They admit continuous linear right inverse operators if and only if

$$\lim_{j\to\infty} \frac{|\operatorname{Im}\lambda_j|}{\omega(\operatorname{Re}\lambda_j)} = 0\,.$$

We also obtain some necessary and sufficient conditions formulated in terms of fundamental solutions of these operators, i. e. the solutions of the equation $S_\mu\nu = \delta$ in $\left(D^p_{\{\omega\}}(\mathbb{R})\right)'$.

Our results continue the investigation of convolution operators in spaces of ultradifferentiable functions and ultradistributions from [1–2]. Theorem 1 is published in [3].

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Evolution variational problems and some related geometric aspects

V. M. Savchin [14]

One of the main properties of the metric tensor is that it completely defines the geometry of the space to which it belongs. The relationship of expressions for the metric tensor and the kinetic energy allows determining the components of the metric tensor of the configuration space of the system by the type of kinetic energy for the system and constructing its geometric model. The subject of the present lecture lies between analytical mechanics, geometry and variational calculus. Tensor methods have long been applied in the dynamics of finite-dimensional systems [7]. They were initially aimed at using the ideas of Riemannian geometry in dynamics. In turn, the problems of mechanics contributed to the development of geometry. Significant results have been obtained over more than a hundred years (see, for example, [1, 2, 4] and references therein). The main objective of the lecture is to identify the relationship between evolution equations with potential operators and geometries of related configuration spaces of the given systems. Using the Hamilton principle, a wide class of such equations is derived. Their structural analysis is carried out, containing operator analogues of the Christoffel symbols of both the 1st and 2nd kind. It is shown that the study of the obtained evolution equations can be associated, in general, with an extended configuration space, the metric of which is determined by the kinetic energy of the given system. In the report there is used the terminology of works [2, 3, 5, 6].

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On topological properties of graphene

A. G. Sergeev [15],

Keywords: graphene, topological invariants.

MSC2020 codes: 81V74

This is a report on the topological properties of graphene. We start from the description of the structure of graphene and its Hamiltonian. We consider next the topological invariant of graphene, called the eigenspace vorticity, introduced by Monaco and Panati and computed for the 1-canonical model.

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Path Integrals and Schwarz Calculus

E. T. Shavgulidze [16]

Keywords: Path integrals; Schwarz integrals; group of diffeomorphisms.

MSC2020 codes: 28C20, 46G12, 47D08

Functional integrals, in particular, Feynman integrals, were first used in the solution of a wide class of problems of quantum mechanics and theoretical physics (see [1]-[4]). The report presents methods for finding Schwarz functional integrals of the form

$$\int F(\varphi)exp\{\int_{S^1} S_\varphi(t)dt + 2\pi^2 n^2 \int_{S^1} (\varphi'(t))^2 dt\} d\varphi$$

on the group of diffeomorphisms of the circle $Diff_+^1(S^1)$, where

$$S_\varphi(t) = \frac{\varphi'''(t)}{\varphi'(t)} - \frac{3}{2}(\frac{\varphi''(t)}{\varphi'(t)})^2$$

is the Schwarzian of a diffeomorphism φ.

Schwarz integrals appear in SYK models, as well as in descriptions of black hole behavior in gravity theory. An example of this type of integral is

$$\int exp\{\frac{1}{\sigma^2}(\int_{S^1} S_\varphi(t)dt + 2\pi^2 \int_{S^1} (\varphi'(t))^2 dt)\} d\varphi = \frac{\sqrt{2\pi}}{\sigma^3} e^{\frac{2\pi^2}{\sigma^2}}$$

which can be explicitly found using transformations related to the group of smooth diffeomorphisms of the unit circle.

Another example of the same type of functional integral is given by the equation

$$\mathcal{E}_\sigma(u,\, v) = \int\limits_{Diff_+^1([0,1])} \delta\left(\varphi'(0) - u\right)\, \delta\left(\varphi'(1) - v\right)\, \mu_\sigma(d\varphi)\,,$$

where the measure

$$\mu_\sigma(d\varphi) = \frac{1}{\sqrt{\varphi'(0)\varphi'(1)}} \exp\left\{ \frac{1}{\sigma^2} \int\limits_0^1 \mathcal{S}_\varphi(\tau) d\tau + \frac{1}{\sigma^2}\left[\frac{\varphi''(0)}{\varphi'(0)} - \frac{\varphi''(1)}{\varphi'(1)}\right] \right\} d\varphi$$

is defined on a group of diffeomorphisms $Diff_+^1([0,\, 1])$.

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On universal C^*-algebras without $*$-polynomial relations

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Keywords: C^*-relation; $*$-polynomial relation; universal C^*-algebra.

MSC2020 codes: 46L05, 46M15, 16R50

Introduction. In the framework of a categorical approach to the notion of a universal C^*-algebra generated by a set of generators subject to relations, T.A. Loring introduced and studied categories called the C^*-relations. Given a set X, a C^*-relation on X is a category whose objects are functions from X to C^*-algebras and morphisms are $*$-homomorphisms of C^*-algebras making the appropriate triangle diagrams commute. Moreover, these functions and $*$-homomorphisms satisfy certain natural axioms. A C^*-relation is said to be compact if it determines a universal C^*-algebra.

C^*-algebras without $*$-polynomial relations. This report is based on the results of the papers [1-4]. We define a C^*-algebraic analogue of the free algebra and describe some properties of this structure. In particular, we will discuss the following results.

Definition. Let X be a set and $\mathcal{R}$ be a compact C^*-relation on X. The C^*-algebra $C^*(\mathcal{R})$ is said to be a <u>universal C^*-algebra without $*$-polynomial relations</u>, if for any non-trivial $*$-polynomial $p(x_1, \ldots, x_k) \in F(X)$ there is an object $f\colon X \to A$ from $\mathcal{R}$ such that $p(f(x_1), \ldots, f(x_k)) \neq 0$, where $F(X)$ is the free $*$-algebra generated by X.

Theorem 1. Let X be a set. Then, for any function $r\colon X \to \mathbb{R}_+\backslash\{0\}$ the C^*-algebra $C^*(X, r)$ is a universal C^*-algebra without $*$-polynomial relations.

Theorem 2. Let X be a set, $P \subset F(X)$ a set of $*$-polynomials which defines a compact C^*-relation $\mathcal{R}(X, P)$, $u\colon X \to C^*(X, P)$ be a corresponding universal representation. Then there is a function $r\colon X \to \mathbb{R}_+\backslash\{0\}$ such that

$$C^*(X, P) \cong C^*(X, r)\Big/P(v),$$

where $v\colon X \to C^*(X, r)$ is a universal representation in the compact C^*-relation $\mathcal{R}(X, r)$. Moreover, for all $x \in X$, one has

$$r(x) = \|u(x)\|$$

and

$$u \cong \pi_P \circ v,$$

where $\pi_P\colon C^*(X, r) \to C^*(X, r)/P(v)$ is the natural projection on the factor algebra and $P(v)$ is the closed two-sided ideal of an algebra $C^*(X, r)$ generated by polynomials $p(v(x_1), \ldots, v(x_k))$, $p \in P$.

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Crystal Length Optimization for SPDC-Generated Photonic Entangled Cat-like States

R. Singh [18], A. E. Teretenkov [19].

Annotation

Building on our prior proposal of photonic Schrodinger cat-like states (SCLSs) in spontaneous parametric down-conversion (SPDC) [arXiv:2405.14526], this work derives the optimal crystal length required for their practical generation. We present analytical and approximate formulas that link crystal and pump parameters directly to the optimal interaction length. These results provide a critical design tool for determining the optimal crystal length, enabling the practical generation of entangled cat states in $\chi^{(2)}$ - nonlinear media.

Keywords: Crystal Length Optimization; SPDC (Spontaneous Parametric Down-Conversion); Photonic Cat-like States; Entangled States

MSC2020 codes: 81V80, 81P40, 81P48

Introduction. Many promising quantum technologies, such as quantum sensing and continuous-variable quantum computing, heavily rely on the ability to generate and control non-classical states of light. Of particular interest are Schrodinger cat states?non-Gaussian states that are quantum superpositions of macroscopically distinguishable states, which also possess the property of quantum entanglement [1]. These states are a key resource for quantum sensing tasks and continuous-variable quantum information protocols.

In our previous work [1], we proposed a method for generating entangled photonic cat-like states via spontaneous parametric down-conversion (SPDC). However, for the practical implementation of this method, it is critically important to determine the optimal experimental parameters, with one of the key parameters being the length of the nonlinear crystal.

Thus, a fundamental question arises: what is the optimal crystal length to optimize the non-classical properties of the generated entangled photonic cat-like states?

In this work, we provide an answer to this question. We present exact and approximate analytical formulas that directly relate crystal parameters (nonlinear susceptibility, refractive index) and pump parameters (wavelength, power) to the optimal interaction length. The optimal crystal length for maximizing the amplitude of cat-like states is determined.

Our result can be applied to advance continuous-variable quantum technologies. It potentially enables the acceleration of practical generation of entangled cat-like states in $\chi^{(2)}$ - nonlinear media, paving the way for their use in:

- Quantum sensing.
- Continuous-variable quantum information protocols.

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Asymptotics of eigenvalues and eigenvectors for large Toeplitz banded matrices

V. A. Stukopin [20],

Keywords: Asymptotic expansion, Toeplitz matrices, spectrum.

MSC2020 codes: 47D08, 35C15, 35J10

Introduction. It is known that explicit formulas for Toeplitz matrices can be expressed in the term of the symbol of initial matrix. Formulas of this kind are most important in the case when the matrix size tends to infinity and not the exact formulas but the asymptotically approximate ones are important.

In the paper [1] were obtain uniform asymptotic formulas for all eigenvalues of symmetric Toeplitz band matrices of large dimension. The entries of the matrices are assumed to be complex, that is, the matrices need not necessarily be self-adjoint. The formulas presented allow a detailed study of the structure of the eigenvalues and of their location in the complex plane, and to build an efficient algorithm for finding their numerical values for matrices of medium and high dimension. In the paper [2] the same formulas were obtained for component of eigenvectors.

We find uniform asymptotic formulas for all the eigenvalues of certain 7-diagonal symmetric Toeplitz matrices of large dimension. The entries of the matrices are real and we consider the case where the real-valued generating function such that its first five derivatives at the one endpoint of interval are equal zero. This is not the simple-loop case considered earlier in [1-2]. We obtain nonlinear equations for the eigenvalues. It should be noted that our equations have a more complicated structure than the equations for the simple loop case. We find uniform asymptotic formulas for all the eigenvalues of certain 7-diagonal symmetric Toeplitz matrices of large dimension ([3]). We also consider the same problem for eigenvectors.

Main result

Theorem 1.

Let $\lambda = g(\varphi)$. Then the equation $\det T_n(a-g(\varphi)) = 0$ is equivalent to the following set of equations:

$$\varphi = \frac{2}{n+3}\left[\pi j + \arctan f(\varphi, n)\right], \tag{1}$$

$$j \in \left\{1, 2, \dots, \left[\frac{n+1}{2}\right]\right\}$$

and

$$\varphi = \frac{2}{n+3}\left[\pi j + \frac{\pi}{2} - \arctan h(\varphi, n)\right], \tag{2}$$

$$j \in \left\{1, 2, \dots, \left[\frac{n}{2}\right]\right\}.$$

where

$$\begin{aligned} f(\varphi, n) &= C_1(\varphi)\tan\left(\frac{n+3}{2}\gamma\right) - C_2(\varphi)\tan\left(\frac{n+3}{2}\beta\right), \\ h(\varphi, n) &= C_1(\varphi)\frac{1}{\tan\left(\frac{n+3}{2}\gamma\right)} - C_2(\varphi)\frac{1}{\tan\left(\frac{n+3}{2}\beta\right)} \end{aligned} \tag{3}$$

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Remark 1. We obtain explicit asymptotic formulas for the eigenvalues ??using the fixed point method based on the obtained equations. These formulas are quite cumbersome.

*Remark 2.*The same result we obtain also for eigenvectors.

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Parametric approximation and perturbative corrections to it

A. E. Teretenkov [21].

Keywords: parametric approximation; open quantum systems; Jaynes–Cummings model; time-convolutionless master equation; non-Markovian dynamics.

MSC2020 codes: 81S22, 81Q15

Introduction. The talk discusses mathematical aspects of parametric approximation in quantum optics. It investigates the reduced dynamics of a two-level system interacting with a dissipative bosonic mode without assuming classicality of the field. It is shown [1] that the standard parametric approximation arises as the first nonzero term in the expansion in the small interaction parameter, while higher-order corrections describe dissipation and non-Markovian effects. It is established that at long times the dynamics reduces to a Markovian semigroup description with effective renormalization of the initial state.

Problem setting. Consider the dissipative Jaynes–Cummings model on $\mathcal{H}_A \otimes \mathcal{H}_B$,

$$\dot{\rho}(t) = \big(\mathcal{L}_0 + \lambda\mathcal{L}\big)\rho(t), \qquad \mathcal{L}_0 = -i[H_A + H_B, \cdot] + \mathcal{D}, \quad \mathcal{L} = -i[H_I, \cdot],$$

with

$$H_A = \omega_A \sigma^+\sigma^-, \quad H_B = \omega_c b^\dagger b, \quad H_I = g\sigma^+ b + \overline{g}\,\sigma^- b^\dagger, \quad \mathcal{D} = \gamma\Big(b \,\cdot\, b^\dagger - \tfrac{1}{2}\{b^\dagger b, \cdot\}\Big),$$

and let $\Delta\omega = \omega_A - \omega_c$, $\Gamma = -i\Delta\omega + \frac{\gamma}{2}$. Assume the Argyres–Kelley type projector

$$\mathcal{P}X = \mathrm{tr}_B(X) \otimes |z\rangle\langle z|, \qquad b|z\rangle = z|z\rangle,$$

and a consistent initial state $\rho(0) = \rho_A(0) \otimes |z\rangle\langle z|$.

Main result.

Theorem 1. The reduced state $\rho_A(t) = \mathrm{tr}_B\, \rho(t)$ admits a time-convolutionless (TCL) perturbative generator expansion

$$\frac{d}{dt}\,\rho_A(t) = \sum_{n \geq 0} \lambda^n \mathcal{K}_n(t)\rho_A(t)$$

as $\lambda \to 0$, whose first nontrivial terms have the following explicit structure:

$$\mathcal{K}_1(t)\rho_A = -i\, e^{-\frac{\gamma}{2}t}\Big[gz\, e^{-i\omega_c t}\sigma^+ + \overline{g}\,\overline{z}\, e^{i\omega_c t}\sigma^-,\, \rho_A\Big],$$

$$\mathcal{K}_2(t)\rho_A = \frac{-i|g|^2}{2|\Gamma|^2}\Big(2\Delta\omega + \varphi_2(t)\Big)\big[\sigma^+\sigma^-, \rho_A\big] + \frac{|g|^2}{|\Gamma|^2}\Big(\gamma + f_2(t)\Big)\Big(\sigma^-\rho_A\sigma^+ - \tfrac{1}{2}\{\sigma^+\sigma^-, \rho_A\}\Big),$$

where

$$f_2(t) = e^{-\frac{\gamma}{2}t}\Big(2\Delta\omega\sin(\Delta\omega t) - \gamma\cos(\Delta\omega t)\Big), \qquad \varphi_2(t) = e^{-\frac{\gamma}{2}t}\Big(-2\Delta\omega\cos(\Delta\omega t) - \gamma\sin(\Delta\omega t)\Big).$$

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Bivariational Approaches to the Numerical Approximation of Dissipative Systems

P. T. Trinh [22].

Keywords: approximate solutions, discretization, Hamilton's action, Ritz method, non-Euler functionals.

The formulations of numerous problems lead to non-potential operators, where the absence of universal algorithms for exact solutions necessitates the use of approximation methods. While variational analysis typically relies on the existence of a corresponding Hamilton's action, it has been shown that such actions may not exist within the framework of standard Euler functionals [1]. However, extending the class of functionals enables the derivation of variational formulations for the original boundary value problem. This expansion allows for the existence of multiple distinct functionals (bivariationality) corresponding to the same problem, each containing complete information about the system.

The present work demonstrates the practical application of these non-Euler functionals for constructing approximate models. A numerical implementation of the Ritz method for a linear ordinary differential equation with a non-potential operator is given, confirming the possibility of obtaining approximate solutions with a high degree of accuracy. Furthermore, a variational approach is utilized to construct two different difference schemes for a specific dissipative problem. A comparison between analytical and approximate solutions is carried out, validating the effectiveness of using extended functionals in problems that admit bivariational formulations [2].

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On Constructing Certain Semigroups of Random Processes in Hilbert Space

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Keywords: bi-continuous semigroup; Chernoff approximation.

MSC2020 codes: 81P20, 60J60, 47D99

Introduction. The report will discuss an approach to constructing Markov semigroups of random processes in Hilbert space associated with quantum Markov processes. In the language of Markov operators, the approximation of semigroups of such processes by discrete-time processes defined by compositions of independent identically distributed random operators is carried out using the Chernoff theorem. Limiting ourselves to subspaces of $C_b(H)$, we discuss the convergence of Chernoff iterations to a bi-continuous semigroup in a sense weaker than in the norm topology.

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On the Problem of Constructing a Measure on a Group of Diffeomorphisms of a Circle

M. B. Vavilov [24], I. A. Bogatyrev [25], I. A. Sukharev [26], E. T. Shavgulidze [27].

Keywords: Wiener measure; Markov process; sub-Laplacian; diffeomorphism group; parabolic equation.

MSC2020 codes: 46F25, 46T12, 46G12

Introduction. In this research, we study one of the ways to construct a measure on trajectories obtained by transferring the Wiener process to the diffeomorphism group $Diff^+(S^1)$ of a circle. Due to the lack of local compactness, there is no Haar measure on the group. Therefore, the following method is considered: constructing a Wiener process on a Lie algebra and lifting it to a group by an exponential map.

The transfer of a two-dimensional Wiener process based on the vectors $\sin(\theta)\dfrac{d}{d\theta}$ and $\sin(2\theta)\dfrac{d}{d\theta}$ is considered. Let us formally write the transfer equation by identifying $S^1 = \mathbb{R}/\mathbb{Z}$:

$$dy_\theta(t) = \sin y_\theta(t) \cdot dW_t^1 + \sin 2y_\theta(t) \cdot dW_t^2 \tag{1}$$

with the initial condition $y_\theta(0) = \theta$.

We will consider diffeomorphisms as elements of the Sobolev space on a segment

$$H = \left\{ h \in C_0[0, 2\pi] \;\middle|\; h(t) = \int_0^t \dot{h}(s)\, ds,\ \dot{h} \in L^2[0, 1] \right\}$$

with a scalar product

$$\langle h_1, h_2 \rangle_H = \int_0^{2\pi} \dot{h}_1(s) \dot{h}_2(s)\, ds.$$

Main results.

Theorem 1. The solution of an infinite-dimensional stochastic differential equation

$$y(t) = f + \int_0^t B_1\big(y(s)\big)\, dW_1(s) + \int_0^t B_2\big(y(s)\big)\, dW_2(s), \tag{2}$$

where

$$B_1(u)(x) = \sin u(x),\ \ B_2(u)(x) = \sin\left(2u(x)\right),$$

in a Banach space

$$S := \{ y \colon [0, T] \times \Omega \to H \mid \|y\|_S < \infty \},$$

where

$$\|y\|_S \;=\; \left(\mathbb{E}\left[\sup_{0 \le t \le T} \|y(t)\|_H^2 \right] \right)^{1/2}$$

exists, is unique, and has continuous trajectories $t \mapsto y(t) \in H$ on a certain segment $[0, T]$.

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It is proved that the solutions of this transfer equation are diffeomorphisms of a circle under certain conditions. It is also proved that the process under consideration has the Markov property, therefore, a sub-Laplacian can be written for it (it is written after certain transformations):

$$
\begin{aligned}
(\Delta_{sub} v)[f] = &\int\limits_{S^1}\int\limits_{S^1} \frac{\delta^2 v}{\delta f(\theta_1)\delta f(\theta_2)} \sin f(\theta_1) \sin f(\theta_2) d\theta_1 d\theta_2 + \int\limits_{S^1} (f')^2 \cos f d\theta \int\limits_{S^1} \frac{\delta v}{\delta f(\theta)} \sin f(\theta) d\theta - \\
&- v \int\limits_{S^1} (f'(\theta))^2 \cos^2 f(\theta) d\theta + \frac{v}{4}\left(\int\limits_{S^1} (f'(\theta))^2 \cos f(\theta) d\theta \right)^2 + \\
&+ \int\limits_{S^1}\int\limits_{S^1} \frac{\delta^2 v}{\delta f(\theta_1)\delta f(\theta_2)} \sin 2f(\theta_1) \sin 2f(\theta_2) d\theta_1 d\theta_2 + 4\int\limits_{S^1} (f')^2 \cos 2f d\theta \int\limits_{S^1} \frac{\delta v}{\delta f(\theta)} \sin 2f(\theta) d\theta - \\
&- 4v \int\limits_{S^1} (f'(\theta))^2 \cos^2 2f(\theta) d\theta + 4v \left(\int\limits_{S^1} (f'(\theta))^2 \cos 2f(\theta) d\theta \right)^2
\end{aligned}
\tag{3}
$$

The corresponding equation of the parabolic type is written (similar to the heat equation) and its solution can be represented as a functional integral with the Shavgulidze measure.

The physical interpretation of the obtained results is contained in the Euler equation for the diffeomorphism group

$$
\begin{aligned}
\frac{\partial u(t,\theta)}{\partial t} = &-2u\cos(\theta) \int\limits_{S^1} u \sin(\theta)\, d\theta - 4u\cos(2\theta) \int\limits_{S^1} u\sin(2\theta)\, d\theta - \\
&- \frac{\partial u}{\partial \theta} \sin(\theta) \int\limits_{S^1} u\sin(\theta)\, d\theta - \frac{\partial u}{\partial \theta} \sin(2\theta) \int\limits_{S^1} u \sin(2\theta)\, d\theta,
\end{aligned}
\tag{4}
$$

obtained using Arnold's theorem.

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Quantum mechanics in real Kahler space

I. Volovich [28]

Quantum mechanics is traditionally formulated in a complex Hilbert space.
However, it can be reformulated in a real Kahler space.
This reformulation allows us to obtain, in particular, the following results:

1) prove the equivalence of the Schrodinger equation and the Euler-Bernoulli equation for describing the vibration of beams and plates in elasticity theory;

2) prove ergodicity for almost all finite-dimensional quantum systems;

3) allow us to propose symplectic computers as an alternative to quantum computers.

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О решениях обыкновенных дифференциальных уравнений с потенциальными операторами

С. А. Будочкина [29]

Рассматривается обыкновенное дифференциальное уравнение третьего порядка

$$N(u) \equiv u'''(t) + a(t)u''(t) + b(t)u'(t) + c(t)u(t) = 0, \qquad t \in (t_0, t_1]. \tag{4}$$

Здесь $u(t)$ – неизвестная функция, $a \in C^2[t_0, t_1]$, $b \in C^1[t_0, t_1]$, $c \in C[t_0, t_1]$ – заданные функции.
Положим

$$D(N) = \left\{ u \in U = C^3[t_0, t_1] : u(t_0) = u_0,\ u'(t_0) = u'_0,\ u''(t_0) = u''_0 \right\}, \tag{5}$$

где u_0, u'_0, u''_0 – заданные постоянные.
Введем билинейную форму

$$\Phi(v, g) = \int_{t_0}^{t_1} v(t)g(T - t)dt, \tag{6}$$

где $T = t_0 + t_1$.
1. Исследована потенциальность оператора N вида (1) на множестве $D(N)$ (2) относительно билинейной формы (3).
2. Найдено решение задачи (1), (2) в случае потенциальности оператора N.

В докладе будут использоваться обозначения и терминология работы [1].

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Новый тип квазифейнмановских формул для параболического уравнения на прямой

А.В. Веденин [30]

В докладе рассмотрена полугруппа операторов, дающая решения задачи Коши для линейного параболического дифференциального уравнения в частных производных с переменными вещественными коэффициентами на вещественной прямой. Предложены черновские аппроксимации этой полугруппы, дающие скорость сходимости не хуже, чем единица, деленная на квадрат номера аппроксимации. Функция Чернова является суммой двух интегралов, поэтому решение уравнения представляется в виде квазифейнмановской формулы.

Символом $UC_b(\mathbb{R})$ всюду далее обозначим банахово пространство всех ограниченных равномерно непрерывных функций $f\colon \mathbb{R} \to \mathbb{R}$, наделённое нормой $\|f\| = \sup_{x\in\mathbb{R}} |f(x)|$. Пусть $a, a', a'', b, b', b'', c, c', c'' \in UC_b(\mathbb{R})$ и $\inf_{x\in\mathbb{R}} a(x) > 0$. Зададим плотно определённый в $UC_b(\mathbb{R})$ оператор H равенством $(H\varphi)(x) = a(x)\varphi''(x) + b(x)\varphi'(x) + c(x)\varphi(x)$ для всех $x \in \mathbb{R}$. Для оператора H построена функция Чернова S, заданная для всех $x \in \mathbb{R}$, $t \geq 0$, $f \in UC_b(\mathbb{R})$ равенством

$$
\begin{aligned}
(S(t)f)(x) &= \\
&= \int_{-1}^{1} \frac{1}{2}\Big(1 + c(x)t\Big) f\Big(x + (6a(x)t)^{\frac{1}{2}} y + b(x)t\Big) dy + \int_{-1}^{1} \left(\sum_{k=0}^{6} \beta_k(t,x) y^k\right) f(x + yt^{\frac{1}{4}}) dy,
\end{aligned}
$$

где

$$
\begin{aligned}
\beta_0(t,x) = \frac{1225}{1024}\Big(a(x)c''(x) + b(x)c'(x) + c(x)^2\Big)t^2 - \\
- \frac{11025}{512}\Big(a(x)a''(x) + 2a(x)b'(x) + b(x)a'(x)\Big)t^{3/2} + \frac{14553}{64}a(x)^2 t,
\end{aligned}
$$

$$
\beta_1(t,x) = \frac{3675}{256}\Big(a(x)b''(x) + 2a(x)c'(x) + b(x)b'(x)\Big)t^{7/4} - \frac{19845}{32}a(x)a'(x)t^{5/4},
$$

$$
\begin{aligned}
\beta_2(t,x) = -\frac{11025}{1024}\Big(a(x)c''(x) + b(x)c'(x) + c(x)^2\Big)t^2 + \\
+ \frac{178605}{512}\Big(a(x)a''(x) + 2a(x)b'(x) + b(x)a'(x)\Big)t^{3/2} - \frac{280665}{64}a(x)^2 t,
\end{aligned}
$$

$$
\beta_3(t,x) = -\frac{6615}{128}\Big(a(x)b''(x) + 2a(x)c'(x) + b(x)b'(x)\Big)t^{7/4} + \frac{42525}{16}a(x)a'(x)t^{5/4},
$$

$$
\begin{aligned}
\beta_4(t,x) = \frac{24255}{1024}\Big(a(x)c''(x) + b(x)c'(x) + c(x)^2\Big)t^2 - \\
- \frac{467775}{512}\Big(a(x)a''(x) + 2a(x)b'(x) + b(x)a'(x)\Big)t^{3/2} + \frac{800415}{64}a(x)^2 t.
\end{aligned}
$$

$$
\beta_5(t,x) = \frac{10395}{256}\Big(a(x)b''(x) + 2a(x)c'(x) + b(x)b'(x)\Big)t^{7/4} - \frac{72765}{32}a(x)a'(x)t^{5/4},
$$

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$$\beta_6(t,x) = -\frac{15015}{1024}\Big(a(x)c''(x) + b(x)c'(x) + c(x)^2\Big)t^2 + \\ + \frac{315315}{512}\Big(a(x)a''(x) + 2a(x)b'(x) + b(x)a'(x)\Big)t^{3/2} - \frac{567567}{64}a(x)^2 t.$$

Функция Чернова S позволяет построить черновские аппроксимации к C_0-полугруппе $(e^{tH})_{t\geq 0}$, которая дает решение к задаче Коши для линейного параболического дифференциального уравнения в частных производных с переменными вещественными коэффициентами на вещественной прямой. А именно, задача Коши

$$\begin{cases} u_t'(t,x) = a(x)u_{xx}''(t,x) + b(x)u_x'(t,x) + c(x)u(t,x), \quad x \in \mathbb{R}, t \geq 0 \\ u(0,x) = u_0(x). \quad x \in \mathbb{R} \end{cases}$$

имеет решение u, представимое в виде $u(t,x) = (e^{tH}u_0)(x) = \lim_{n\to\infty} u_n(t,x)$, где черновские аппроксимации задаются равенством $u_n(t,x) = (S(t/n)^n u_0)(x)$, в котором символы $S(t/n)^n$ обозначают композицию n копий линейного ограниченного оператора $S(t/n)$, действующего в $UC_b(\mathbb{R})$.

Функция Чернова S является суммой двух интегралов по отрезку $[-1,1]$, поэтому функция u_n записывается как сумма 2^n интегралов по кубу $[-1,1]^n$, причём каждый из этих интегралов уже записан как повторный (n-кратный) интеграл по отрезку $[-1,1]$. Таким образом, решение уравнения представляется в виде квазифейнмановской формулы (ср. [1-3]) нового вида.

Более того, если коэффициенты a, b, c имеют 7 ограниченных производных, а начальное условие u_0 имеет 9 ограниченных производных, то предложенные черновские аппроксимации u_n дают скорость сходимости к решению u не хуже, чем константа, деленная на квадрат номера аппроксимации. Это доказывается с помощью теоремы Галкина-Ремизова [4].

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Энтропия по Больцману и Пуанкаре и эргодическая теория

В.В.Веденяпин [31]

Ключевые слова: H-теорема, энтропия Больцмана, энтропия Пуанкаре, эргодическая проблема, круговая модель Каца, клеточные автоматы.

Предлагаются новые формы H-теоремы, обобщающие классические теоремы Больцмана и Пуанкаре. Предлагается теорема: временные средние совпадают с экстремалями Больцмана. Эта теорема сводит поиск предельной функции к поискам интегралов, устанавливает связь между обратимостью гамильтоновых систем и необратимостью роста энтропии. Для круговой модели Каца размерность пространства линейных законов сохранения связана с малой теоремой Ферма. Обсуждается модель Каца как пример клеточного автомата.

H-теорема впервые была рассмотрена Больцманом в работе "Weitere Studien ?uber das W?armegleichgewicht unter Gasmolek?ulen" (1872) [1]. Эту теорему, обос-новывающую сходимость решений уравнений типа Больцмана к максвелловскому распределению, Больцман связал с законом возрастания энтропии. Мы обобщаем эту H-теорему в различных направлениях, доказывая ее как для уравнений типа химической кинетики в классическом и квантовом случаях, а также к уравнениям Лиувилля. Теоремы Лиувилля или неразрывности – это область эргодической теории , и эти новые формы H-теоремы были найдены в 1906 году А.Пуанкаре и обобщены и обоснованы в 2001-2003 годах В.В.Козловым и Д.В.Трещевым. Предлагаются новые формы H-теоремы, обобщающие классические теоремы Больцмана и Пуанкаре и теоремы Козлова-Трещева.. Предлагается теорема: временные средние совпадают с экстремалями Больцмана. Эта теорема сводит поиск предельной функции к поискам интегралов, устанавливает связь между обратимостью гамильтоновых систем и необратимостью роста энтропии. Для круговой модели Каца размерность пространства линейных законов сохранения связана с малой теоремой Ферма. Обсуждается модель Каца как пример клеточного автомата.[3-4]

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Схема доказательства теоремы о скорости сходимости черновских аппроксимаций

О.Е.Галкин,[32] **И.Д.Ремизов**[33]

В докладе будеть представлена схема доказательства теоремы о скорости сходимости черновских аппроксимаций из работы [1]. Также будет рассказано о выражении решения линейного ОДУ с переменными коэффициентами через эти коэффициенты по методу из работы [2].

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Частичные пределы аппроксимаций Чернова

Р.Ш. Кальметьев

Исследуется предельное поведение аппроксимаций Чернова для операторных полугрупп, когда классические условия теоремы Чернова выполняются не полностью. Рассматривается пример, в котором замыкание производной аппроксимирующей операторной функции Чернова в нуле является симметричным оператором с конечными и равными индексами дефекта.

Дробное исчисление Ито для случайно масштабированного дробного броуновского движения и его приложения к эволюционным уравнениям.

Я.А. Киндеркнехт (Бутко) [34]

Экспериментально хорошо подтвержденная аномальная диффузия — это явление, наблюдаемое во множестве различных природных систем, принадлежащих к разным областям исследований. В частности, аномальная диффузия стала фундаментальным понятием в изучении живых биологических систем. В общем смысле, термин «аномальная диффузия» обозначает все те диффузионные процессы, которые подчиняются законам, отличным от законов классической диффузии, а именно все случаи, когда смещения частиц не описываются гауссовой функцией плотности вероятности и/или дисперсия таких смещений не растет линейно со временем. Одной из распространённых моделей аномальной диффузии является случайно масштабированное (или суперстатистическое) дробное броуновское движение.

В докладе представлены результаты работы [1], в которой определяется дробный стохастический интеграл Ито относительно случайно масштабированного дробного броуновского движения, доказывается формула Ито для функций от таких стохастических интегралов, показано, что построенные стохастические интегралы служат стохастическими решениями для широкого класса интегро-дифференциальных эволюционных уравнений, содержащих нелокальные операторы, действующие по временной переменной. Наши результаты охватывают такие частные случаи случайно масштабированного дробного броуновского движения, как гамма-серое броуновское движение, обобщенное серое броуновское движение, суперстатистическое дробное броуновское движение со случайным коэффициентом диффузии, имеющим обобщенное гамма-распределение, а также некоторые другие процессы, обсуждаемые в литературе.

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Что есть жизнь? Математический аспект

С. В. Козырев [35]

Keywords: мультиагентные системы; роевой интеллект.

MSC2020 codes: 93A16

Обсуждается определение биологических систем, восходящее к работам Гельфанда и Цетлина в 60-х годах прошлого века. Такое определение описывает систему взаимодействующих агентов, действующих в организованном мире и организованных в синергии. Обсуждаются примеры синергий в биологии, теории обучения и генеративном искусственном интеллекте.

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Теория приближений в дробном анализе

В. Н. Колокольцов [36] [37] [38]

Аннотация. В докладе будет дан обзор недавних результатов автора и его коллег по методам приближений сложных объектов дробного анализа более простыми (легче поддающимся вычислениям): в частности, дробных (и обобщенных дробных) производных и интегралов матрицами, а полумарковских процессов, описываемых дробными уравнениями, — случайными блужданиями в непрерывном времени.

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Функциональные интегралы и методы численного интегрирования

А. А. Лобода [39]

Аннотация. В докладе будут рассмотрены различные функциональные интегралы, в том числе и являющиеся представлениями решений дифференциальных уравнений, таких, как уравнения теплопроводности с различными потенциалами, уравнения Шрёдингера и стохастические уравнения. Для некоторых из интегралов будут указаны квадратурные формулы, способы численного интегрирования с помощью построения конечнократных приближений. Будут кратко разобраны возможности применить методы Монте-Карло.

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Предельные точки итераций Чернова и соответствующие фейнмановские меры

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Определена и исследована биекция множества цилиндрических процессов в евклидовом пространстве E на пространство операторнозначных функций, отображающих вещественную полуось в пространство $B(L_2(E))$. Для возмущений генератора C_0-полугруппы в пространстве $L_2(E)$ ограниченным оператором умножения на функцию получено представление Фейнмана-Каца возмущенной C_0-полугруппы интегралом от функционала возмущения на пространстве траекторий по цилиндрической мере, которую биекция сопоставляет невозмущенной полугруппе.

Исследована сходимость и компактность фейнмановских мер, соответствующих итерациям функции Чернова. Показано, что при выполнении «почти всех» условий теоремы Чернова последовательной фейнмановских мер расходится, но компактна. Получено описание множества ее частичных пределов. Мера, индуцированная на множестве частичных пределов мер Фейнмана, представлена как мера Фейнмана на траекториях в расширенном пространстве.

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Квазиклассическое квантование векторных расслоений над окружностью и спектральные серии одномерного оператора Шредингера с двухмасштабным потенциалом

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Теория В.П. Маслова позволяет описывать спектральные серии широкого класса дифференциальных операторов с гладкими коэффициентами в терминах геометрических объектов - ларганжевых подмногообразий классического фазового пространства. Если коэффициенты содержат сингулярности, соответствующие геометрические конструкции должны модифицироваться. Мы опишем такую модификацию для оператора Шредингера, потенциал которого испытывает сглаженный скачок (т.е. его слабый предел по квазиклассическому параметру - разрывная функция). В зависимости от ширины скачка, спектральные серии соответствуют различным геометрическим объектам - либо негладким кривым, либо векторным расслоениям над окружностью.

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